University of Debrecen
Faculty of Science and Technology
Institute of Mathematics

BSc Program

MATHEMATICS

Mathematics BSc

Basics

Code	Subject	Credit	Hours/week		Examination	Prerequisites	Semester
			Theory	Practice			
TTMBG0001	Basics of mathematics	0		1	S		1
TTMBE0101	Introduction to algebra and number theory	3	2		E	TTMBG0101(p)	1
TTMBG0101	Introduction to algebra and number theory	3		3	P		1
TTMBE0102	Linear algebra 1.	3	2		E	TTMBG0102(p)	1
TTMBG0102	Linear algebra 1.	2		2	P		1
TTMBE0103	Linear algebra 2.	3	2		E	TTMBE0102 TTMBG0103(p)	2
TTMBG0103	Linear algebra 2.	2		2	P	TTMBE0102	2
TTMBE0104	Algebra 1.	3	2		E	TTMBE0101 TTMBE0102 TTMBG0104(p)	2
TTMBG0104	Algebra 1.	2		2	P	TTMBE0101 TTMBE0102	2
TTMBE0105	Algebra 2.	3	2		E	TTMBE0104 TTMBG0105(p)	3
TTMBG0105	Algebra 2.	2		2	P	TTMBE0104	3
TTMBE0106	Number theory	3	2		E	TTMBE0104 TTMBG0106(p)	3
TTMBG0106	Number theory	2		2	P	TTMBE0104	3
TTMBE0107	Combinatorics and graph theory	4	3		E	TTMBG0107(p)	1
TTMBG0107	Combinatorics and graph theory	2		2	P		1
TTMBE0201	Sets and functions	3	2		E	TTMBG0201(p)	1
TTMBG0201	Sets and functions	2		2	P		1
TTMBE0202	Introduction to analysis	4	3		E	TTMBE0201 TTMBG0202(p)	2
TTMBG0202	Introduction to analysis	2		2	P	TTMBE0201	2
TTMBE0203	Differential and integral calculus	4	3		E	TTMBE0202 TTMBG0203(p)	3
TTMBG0203	Differential and integral calculus	3		3	P	TTMBE0202	3
TTMBE0204	Differential and integral calculus in several variables	4	3		E	TTMBE0203 TTMBG0204(p)	4
TTMBG0204	Differential and integral calculus in several variables	3		3	P	TTMBE0203	4
TTMBE0205	Measure and integral theory	3	2		E	TTMBE0203	4
TTMBE0206	Ordinary differential equations	3	2		E	TTMBE0204 TTMBG0206(p)	5
TTMBG0206	Ordinary differential equations	2		2	P	TTMBE0204	5
TTMBE0301	Geometry 1.	3	2		E	TTMBG0301(p)	1
TTMBG0301	Geometry 1.	2		2	P		1
TTMBE0302	Geometry 2.	3	2		E	TTMBE0102 TTMBG0302(p)	2
TTMBG0302	Geometry 2.	2		2	P	TTMBE0102	2

TTMBE0303	Differential geometry	3	2		E	TTMBE0302 TTMBE0204 TTMBG0303(p)	5
TTMBG0303	Differential geometry	2		2	P	TTMBE0302 TTMBE0204	5
TTMBE0304	Vector analysis	3	2		E	TTMBE0204 TTMBG0304(p)	6
TTMBG0304	Vector analysis	2		2	P	TTMBE0204	6
TTMBE0401	Probability theory	4	3		E	TTMBE0205 TTMBG0401(p)	5
TTMBG0401	Probability theory	2		2	P	TTMBE0205	5
TTMBE0402	Statistics	4	3		E	TTMBE0401 TTMBG0402(p)	6
TTMBG0402	Statistics	2		2	P	TTMBE0401	6
TTMBG0601	Introduction to informatics	2		3	P		1
TTMBG0602	Programming languages	2		2	P		2
	Basic information	0		1	S		1

Advanced prof module

Code	Subject	Credit	Hours/week		Examination	Prerequisites	Semester
			Theory	Practice			
TTMBE0109	Applied number theory	3	2		E	TTMBE0106	4
TTMBG0110	Algorithms in algebra and number theory	3		3	P	TTMBE0106	4
TTMBE0111	Introduction to cryptography	3	2		E	TTMBE0109 TTMBG0111(p)	5
TTMBG0111	Introduction to cryptography	2		2	P	TTMBE0109	5
TTMBE0209	Numberical analysis	4	3		E	TTMBE0102 TTMBE0203 TTMBG0209(p)	4
TTMBG0209	Numberical analysis	2		2	P	TTMBE0102 TTMBE0203	4
TTMBG0210	Analysis with computer	3		3	P	TTMBE0203	6
TTMBE0211	Economic mathematics	3	2		E	TTMBE0204 TTMBG0211(p)	5
TTMBG0211	Economic mathematics	2		2	P	TTMBE0204	5
TTMBG0308	Computer geometry	3		3	P	TTMBE0302	3
TTMBE0606	Algorithms	3	2		E	TTMBE0107 TTMBG0606(p)	2
TTMBG0606	Algorithms	2		2	P	TTMBE0107	2
TTMBE0607	Linear programming	3	2		E	TTMBE0102 TTMBG0607(p)	3
TTMBG0607	Linear programming	2		2	P	TTMBE0102	3
TTMBE0608	Nonlinear optimization	3	2		E	TTMBE0204 TTMBG0608(p)	5
TTMBG0608	Nonlinear optimization	2		2	P	TTMBE0204	5
TTMBG0403	Computer statistics	2		2	P	TTMBE0401	6

Others

Code	Subject	Credit	Hours/week		Examination	Prerequisites	Semester
			Theory	Practice			
TTFBE2211	Classical mechanics	4	2	1	E	TTMBE0203	4
TTFBE2212	Theoretical mechanics	4	2	1	E	TTFBE2211	6

						TTMBE0206	
TTTBE0030	European Union studies	1	1		E		1
TTTBE0040	Basic environmental science	1	1		E		1

Thesis and free optional courses

Code	Subject	Credit	Hours/week		Examination	Prerequisites	Semester
			Theory	Practice			
TTMBG0701	Thesis 1.	5			P	TTMBG0001 TTMBE0101 TTMBE0102 TTMBE0202 TTMBE0301	5
TTMBG0702	Thesis 2.	5			P	TTMBG0701	6
	Free optional courses	9					

E - Exam

P - Practical

S - Signature

Subjects

Basics

TTMBG0001
Basics of mathematics
0+1 classes/week, 0+0 credit, S
Lecturer: Dr. Varga Nóra
Prerequisities: none

Algerbaic transformations. Solution of different type equations, equation systems, inequalities and inequality systems. Basic notions of trigonometry and coordinate geometry.
Compulsory/Recommended Readings:
A. Bérczes and Á. Pintér: College Algebra. University of Debrecen, 2013. http://math.unideb.hu/media/berczes-attila/College-Algebra.pdf
R. D. Gustafson: College algebra and trigonometry. Pacific Grove, Brooks/Cole, 1986.

TTMBE0101, TTMBG0101
Introduction to algebra and number theory
2+3 classes/week, 3+3 credit, E+P
Lecturer: Dr. Pintér Ákos
Prerequisities: none

Relations, algebraic structures, operations and their properties. Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm. Congruence relation and congruence classes in Z, rings of congruence classes. The theorem of Euler-Fermat. Linear conguences. Linear congruence systems, Chinese remainder theorem. Two-variable and multivariate linear Diophantine equations. Peano axioms, N, Z, Q. Complex numbers, operations, conjugate, absolute value. Trigonometric form of complex numbers, theorem of Moivre, n^th roots of complex numbers, roots of unity. Polynomial ring over field. Euclidean division, greatest common divisor. Ring of Z[x], Q[x], R[x], C[x] , absolute value. Fundamental theorem of algebra. Partial fraction expression. Algebraic equaitions, discriminant, resultanat, multiple roots, cubic and quartic equations. Multvariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomail.
Compulsory/Recommended Readings:
I. Nivan, H. S. Zuckerman, H. L. Montgomery: An introduction to the theory of numbers. John Wiley and Sons, 1991 [1960]. http://www.fuchs-braun.com/media/532896481f9c1c47ffff8077fffffff0.pdf
L. N., Childs:: A concrete introduction to higher algebra. New York, Springer, 2000.

TTMBE0102, TTMBG0102
Linear algebra 1.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Gaál István
Prerequisities: none

Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.
Compulsory/Recommended Readings:
Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.
Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.
Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

TTMBE0103, TTMBG0103
Linear algebra 2.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Gaál István
Prerequisities: TTMBE0102

Linear forms, bilinear forms, quadratic forms. Inner product, Euclidean space. Inequalities in Euclidean spaces. Orthonormal bases. Gram-Schmidt orthogonalization method. Orthogonal complement of a subspace. Complex vectorspaces with inner product: unitary spaces. Linear forms, bilinear forms and inner products. Adjoint of a linear transformation. Properties of the adjoint transformation. Selfadjoint transformations. Isometric/orthogonal transformations. Normal transformations.

Compulsory/Recommended Readings:

Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.
Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.
Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

TTMBE0104, TTMBG0104
Algebra 1.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Gábor Horváth
Prerequisites: TTMBE0101, TTMBE0102

Definition of groups, examples. Permutations, sign of permutations. Homomorphisms. Order, cyclic groups. Subgroups, generated subgroups, Lagrange´s theorem. Direct product, the fundamental theorem of finite Abelian groups. Permutation groups and group actions, Cayley´s theorem. Homomorphisms and normal subgroups, conjugation. Factor groups. Homomorphism theorem. Isomorphism theorems. Basic properties of p-groups , center. Definition of rings, examples. Subrings, generated subrings. Finite rings without zero divisors. Homomorphisms and ideals, factor rings. Rings of polynomials. Euclidean rings and principal ideal domains, the fundamental theorem of number theory. Fields, simple algebraic extensions. Minimal polynomial. The multiplicativity formula for degrees. Algebraic numbers. Construction of the splitting field. Characteristics, prime fields. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials over Z_p with arbitrary degree. Geometric constructions with compass and straightedge: The impossibility of doubling a cube (a. k. a. the Delian problem), trisecting an edge and squaring a circle.

Compulsory/Recommended Readings:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.
Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

TTMBE0105, TTMBG0105
Algebra 2.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Gábor Horváth
Prerequisites: TTMBE0104

Sylow´s theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck´s theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert´s basis theorem. Algebras, minimal polynomial over algebras. Frobenius´ theorem. Splitting field, existence, uniquencess, algebraic closure, existence. Normal extensions, extensions of perfect fields are simple. Galois theory. Funadamental theorem of algebra. Compass and straightedge constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

Compulsory/Recommended Readings:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.
Derek J. S. Robinson: A course in the theory of groups, Springer-Verlag, 1980.

TTMBE0106, TTMBG0106
Number theory
2+2 classes/week, 3+2 credit, E+P

Lecturer: Dr. Hajdu Lajos
Prerequisities: TTMBE0104

Orders of elements, generators and their description in Z_p. Quadratic residues modulo p. Residues of higher degree. Arithmetic functions. Additive and multiplicative functions, some important arithmetic functions. Summatory function and Mobius-transform of arithmetic functions. The infinitude of the set of primes. Famous pronblems concerning primes. Primes in arithmetic progressions, the theorem of Dirichlet. The sume of the reciprocals of primes. The $\Pi(x)$ function, the prime number theorem. Lattices, the theorems of Blichfeldt and Minkowski and their applications. The Waring problem. Pithagorean triples. Algebraic numbers, algebraic integers. The field of algebraic numbers and the ring of algebraic integers. Algebraic number fields. Degree, basis, integers and units of a number fields. Quadratic number fields and their representations int he form $Q(\sqrt{d})$.
Compulsory/Recommended Readings:
I. Niven, H. S. Zuckerman, H. L. Mongomery: An Introduction to the Theory of Numbers, Wiley, 1991.
K. Ireland, M. Rosen, A classical introduction to modern number theory (Second edition), Springer-Verlag.
Tom Apostol, Introduction to Analytic Number Theor, Springer-Verlag.

TTMBE0107, TTMBG0107
Combinatorics and graph theory
3+2 classes/week, 4+2 credit, E+P
Lecturer: Dr. Nyul Gábor
Prerequisities: none

Fundamental enumeration problems: permutations, variations, combinations. Properties of binomial coefficients, binomial and multinomial theorem. Inversions, parity, product of permutations, cycles. Inclusion–exclusion principle and applications. Basic definitions of graph theory. Eulerian trail, Hamiltonian path and cycle. Trees and forests, spanning trees, Prüfer code and Cayley's formula. Bipartite graphs. Plane graphs, dual graph, Euler's formula, planar graphs and their characterization. Vertex and edge colourings of graphs, chromatic number, the five color theorem, chromatic polynomial, chromatic index. Fundamentals of Ramsey theory. Matrices of graphs.
Compulsory/Recommended Readings:
Béla Andrásfai: Introductory Graph Theory, Akadémiai Kiadó, 1977.
N. Ya. Vilenkin: Combinatorics, Academic Press, 1971.
Miklós Bóna: A Walk Through Combinatorics, World Scientific, 2017.

TTMBE0201, TTMBG0201
Sets and functions
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Lovas Rezső
Prerequisities: none

Foundations of set theory. Relations. Equivalence and order relations, functions. Basic notions in partially ordered sets and Tarski’s fixed point theorem. Cardinality of sets, Cantor’s theorem and the Schröder–Bernstein theorem. Axioms of the real numbers and their corollaries. Notable subsets of the reals: natural numbers, integers, rational and irrational numbers. Uniqueness of the set of real numbers. Existence and uniqueness of the nth root of a nonnegative number. The p-adic represantion of real numbers. Notable inequalities. The field of complex numbers. Cardinality of sets of numbers.
Compulsory/Recommended Readings:
Walter Rudin: Principles of Mathematical Analysis, McGraw-Hill, New York, 1976.

TTMBE0202, TTMBG0202
Introduction to analysis
3+2 classes/week, 4+2 credit, E+P
Lecturer: Dr. Bessenyei Mihály
Prerequisities: TTMBE0201

Convergence of sequences of real numbers. Relations between convergence, boundedness and monotonicity. The Bolzano–Weierstrass theorem and Cauchy’s criterion for convergence. Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number. Accumulation points, lower and upper limit of sequences. Applications. Convergence of sequences of complex numbers. The Bolzano–Weierstrass-theorem and Cauchy's criterion for sequences of complex numbers. Relations between convergence and the operations. Series of complex numbers; absolute and conditional convergence. Summation of series and operations, grouping and rearranging series. Riemann's theorem. Complex geometric series; the comparison, root and ratio tests. Abel's formula; the theorems of Dirichlet, Leibniz and Abel. Cauchy product, Mertens theorem. Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem. Elementary functions and their addition formulas. Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces. Equivalent metrics and equivalent norms. Hausdorff's criterion for compactness. Special norms of Euclidean spaces. The Bolzano–Weierstrass-theorem and the Heine–Borel theorem. Continuity and its characterization in terms of sequences in metrc spaces. Continuity and operations, the continuity of composite functions. Relations between compactness and continuity, respectively connectedness and conitnuity. Continuous bijections on compact sets. Uniform continuity and its characterization.

Compulsory/Recommended Readings:

W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

E. Hewitt, K. R. Stromberg: Real and Abstract Analysis. Springer-Verlag, 1965.

K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

TTMBE0203, TTMBG0203
Differential and integral calculus
3+3 classes/week, 4+3 credit, E+P
Lecturer: Dr. Bessenyei Mihály
Prerequisities: TTMBE0202

Limit of functions and its computation using limit of sequences. Cauchy's criterions; the relation between the limit and the operations, respectively the order. The relation between limit and uniform convergence, respectively continuity and uniform convergenc; Dini's theorem. Right- and left-sided limits; points of discontinuity; classification of discontinuities of the first kind; limit properties of monotone functions. Elementary limits; the introduction of pi. Functions stemming from elementary functions. Differentiability and approximation with linear functions. Differentiability and continuity; differentiability and operations; the chain rule and the differentiability of the inverse function. Local extremum, Fermat principle. The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules. Higher order differentiability; Taylor's theorem, monotonicity and differentiability, higher order conditions for extrema. Convex functions. The definition of antiderivatives; basic integrals, rules of integration. Riemann integral and criteria for integrability; properties of the integral and methods of integration. The main classes of integrable functions. Inequalities, mean value theorems for the Riemann integral. The Newton–Leibniz theorem and the properties of antiderivatives. The relation between Riemann-integrability and uniform convergence. Lebesgue's criterion. Improper Riemann integral and its criteria.

Compulsory/Recommended Readings:

W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

TTMBE0204, TTMBG0204
Differential and integral calculus in several variables
3+3 classes/week, 4+3 credit, E+P
Lecturer: Dr. Páles Zsolt
Prerequisities: TTMBE0203

Banach's contraction principle. Linear maps. The Fréchet derivative; chain rule, differentiability and operations. The mean value inequality of Lagrange. Inverse and implicit function theorems. Further notions of derivatives; the representation of the Fréchet derivative. Continuous differentiability and continuous partial differentiability; sufficient condition for differentiability. Higher order derivatives; Schwarz–Young theorem, Taylor's theorem. Local extremum and Fermat principle; the second order condition for extrema. The definition of the Riemann integral; the integral and operations, criteria for integrability, inequalities and mean value theorems for the Riemann integral. The relation between the Riemann integral and the uniform convergence. Lebesgue's theorem. Fubini's theorem. Jordan measure and its properties; integration over Jordan measurable sets. Fubini's theorem on simple regions, integral transformation. Functions of bounded variation, total variation, decomposition theorem of Jordan. The Riemann–Stieltjes integral and its properties. Integration by parts. Sufficient condition for Riemann–Stieltjes integrability and the computation of the integral. Curve integral; potential function and antiderivative. Necessary and sufficient conditions for the existence of antiderivatives.

Compulsory/Recommended Readings:
W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

TTMBE0205
Measure and integral theory
2+0 classes/week, 3+0 credit, E
Lecturer: Dr. Nagy Gergő
Prerequisities: TTMBE0203

Measure spaces and measures, their properties. Outer measures, pre-measures. Construction of measures. Lebesgue measure and its topological properties. Borel sets. The structure theorem of open sets. Approximation theorem. The properties of the Cantor set. Existence of non Lebesgue measurable sets. The Lebesgue–Stieltjes measure. Measurable functions and their basic properties, Lusin's theorem. Sequences of measurable functions. heorems of Lebesgue and Egoroff, Riesz's theorem on convergence in measure, approximation lemma. The Lebesgue integral of non-negative measurable functions. Beppo Levi's theorem, Fatou's lemma. The relation between the integral and the sum. Integrable functions. Lebesgue's majorized convergence theorem. The σ-additivity and the absolute continuity of the integral. The Lebesgue integral of complex functions. L^p spaces. Minkowski and Hölder inequality. The Riesz–Fischer theorem. The relation between the Riemann and the Lebesgue integral. Fubini's theorem. The n-dimensional Lebesgue measure. Lebesgue's differentiability theorem. Functions of bounded variation and absolute continuous functions. Basic properties of antiderivatives. The Newton–Leibniz formula.
Compulsory/Recommended Readings:
H. Federer: Geometric Measure Theory. Springer-Verlag, 1969.
Paul R. Halmos: Measure Theory. D. Van Nostrand Company, Inc., New York, 1950.
Anthony W. Knapp: Basic Real Analysis. Birkhauser, Boston-Basel-Berlin, 2005.

TTMBE0206, TTMBG0206
Ordinary differential equations
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Gát György
Prerequisities: TTMBE0204

Differential equations solvable in an elementary way. Cauchy problem; solution, maximal solution, locally and globally unique solution. Lipschitz condition; the theorem on global-local existence and uniqueness. Continuous dependence on the initial value. The Arzela–Ascoli theorem and Peano's theorem. First order linear systems of differential equations; fundamental matrix, Liouville's formula, variation of constants. The construction of fundamental matrices of linear systems of differential equations with constant coefficients. Higher order (linear) differential equations and the Transition Principle; Wronski determinant and Liouville's formula. Fundamental sets of solutions of higher order linear differential equations with constant coefficients. Stability; Gronwall–Bellmann lemma and the stability theorem of Ljapunov. Elements of calculus of variations: the Du Bois-Reymond lemma and the Euler–Lagrange equations. Applications.
Compulsory/Recommended Readings:
E. A. Coddington, N. Levinson: Theory of Ordinary Differential Equations. McGraw-Hill, 1955.

TTMBE0301, TTMBG0301
Geometry 1.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Vincze Csaba
Prerequisities: none

Absolute Geometry: incidence axioms, ruler postulate, plane separation postulate, protractor postulate and the axiom of congruence. Some representative results in Absolute Geometry: congruence theorems, perpendicular and parallel lines, sufficient conditions for parallelism, inequalities. The Euclidean parallel postulate and some equivalent statements. Introduction to the Euclidean geometry (theorems for parallelograms, Intercept theorem and its relatives, similar triangles). Euclidean plane isometries: three mirrors suffice, the classification theorem. The classification of the Euclidean space isometries. Similarities, the fixpoint theorem and the classification of plane/space similarities. The general notion of congruence and similarity. Geometric measure theory: area of polygons, Jordan measure, the area of a circle. The axioms of measuring volumes, the volume of a sphere . The perimeter of a circle, the area of a sphere.

Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098_college_geometry/index.html
John Roe: Elementary Geometry, Oxford University Press, 1993.

TTMBE0302, TTMBG0302
Geometry 2.
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Vincze Csaba
Prerequisities: TTMBE0102

Euclidean-Affin Geometry: vectors. Affine transformations, translations and central similarities. The ratio of three collinear points. Some representative results in Affine Geometry: Ceva's theorem, Menelaus' theorem. Analytic Euclidean-Affine geometry. Linear transformations, the general linear group. The analytic description of affine transformations. The fundamental theorem. Dot and cross product, vector triple product: the geometric characterization and the analytic formulas. Higher dimensional analytic geometry: reflections and isometries of the n-dimensional Euclidean space. The orthogonal group. Lower dimensional cases: two- and three-dimesnional spaces. Coordinate geometry: lines and planes. Implicite and parametric forms. Quadratic curves and surfaces. An introduction to Convex Geometry: convex sets and convex hulls. Carathéodory's theorem. Radon's lemma and Helly's theorem. Convex polygons and polyhedra. Euler's theorem, Descartes' theorem, regular convex polyhedra.

Compulsory/Recommended Readings:
S. R. Lay: Convex Sets and Their Applications, John Wiley & Sons, Inc., 1982.
John Roe: Elementary Geometry, Oxford University Press, 1993.
Csaba Vincze: Convex Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0025, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011_0025_mat_14/index.html

TTMBE0303, TTMBG0303
Differential geometry
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Muzsnay Zoltán
Prerequisities: TTMBE0302, TTMBE0204

Curves in the plane and in space. Curvature of plane curves. Curvature and torsion of space curves. Classification of curves in Euclidean space. Surfaces in the Euclidean space. The tangent plane; the differential of a map. The first and second fundamental forms. Normal an principal curvatures. Gauss and Minkowski curvature. The compatibility equations and the Gauss theorem. Parallel transport along curves. Variation of arc length and energy. Geodesics. Minimizing property of geodesics. The Gauss-Bonnet Theorem. Surfaces of constant curvature.
Compulsory/Recommended Readings:
M. DoCarmo: Differential Geometry of Curves and Surfaces, Prentice-Hall, New-Jersey, 1976.
M. Spivak: A Comprehensive Introduction to Differential Geometry, Vol. II, Publish or Perish, Inc.

TTMBE0304, TTMBG0304
Vector analysis
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Vincze Csaba
Prerequisities: TTMBE0204

Scalar fields: level curves and surfaces. The gradient and its geometric interpretation. Vector fields, the invariants of the Jacobian matrix: divergence and rotation (the vector invariant of the skew-symmetric part of the Jacobian). The Laplace operator. Parametrized curves, line integrals and work done. Stokes' theorem and its applications in the plane: conservative vector fields and potential (path independence for line integrals, rotation-free vector fields, exact differential equations). Parametrized surfaces, surface integrals: the fluxus. The Gauss-Ostrogradsky theorem and the Stokes' theorem in the space. Divergence and flux density. Rotation and circulation density. Identities and computational rules for vector operators: gradient, divergence and rotation. The derivative of the determinant function: the special linear group and its Lie algebra. The orthogonal group and its Lie algebra. Displacement fields: strain and rotational tensors. Integral curves and flows. Divergence-free vector fileds (Liouville theorem, incompressible flows). Harmonic, subharmonic and superharmonic functions, the maximum principle.

Compulsory/Recommended Readings:
M. H. Protter, H. F. Weinberger: Maximum Principles in Differential Equations, Springer New York, 1984.
E. C. Young: Vector and Tensor Analysis, New York : M. Dekker, 1978.

TTMBE0401, TTMBG0401
Probabilty theory
3+2 classes/week, 4+2 credit, E+P
Lecturer: Dr. Fazekas István
Prerequisities: TTMBE0205

Probability, random variables, distribution. Asymptotic theorems of probability theory.
Compulsory/Recommended Readings:
A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.

TTMBE0402, TTMBG0402
Statistics
3+2 classes/week, 4+2 credit, E+P
Lecturer: Dr. Barczy Mátyás
Prerequisities: TTMBE0401

Random samples, estimation, hypothesis testing, analysis of variance, regression analysis.
Compulsory/Recommended Readings:
A. A. Borovkov: Matematical statistics, CRC Press, 1999.
W. R. Pestman, I. B. Alberink: Mathematical Statistics – Problems and detailed solutions, Walter de Gruyter, 1998.

TTMBG0601
Introduction to informatics
0+3 classes/week, 0+2 credit, P
Lecturer: Dr. Tengely Szabolcs
Prerequisities: none

An introduction to LaTeX, a document preparation system for high-quality typesetting. Typesetting of complex mathematical formulas in LaTeX.
Presentation creation using the Beamer class. Writing a formal or business letter in LaTeX. Using the moderncv class for typesetting curricula vitae.
The memoire class, a tool to creat BSc/MSc thesis. Introduction to SageMath, a computer algebra package. The Jupyter Notebook interface and the SageMathCloud.
Basic tools, assignment, equality, and arithmetic. Boolean expressions, loops, lists and sets. Writing functions in SageMath.
Compulsory/Recommended Readings:
T. Oetiker: The Not So Short Introduction to LaTeX
Gregory Bard: SageMath for Undergraduates (http://www.gregorybard.com/Sage.html)

TTMBG0602
Programming languages
0+2 classes/week, 0+2 credit, P
Lecturer: Dr. Bazsó András
Prerequisities: none

Classification of programming languages. Main differences of high-level programming languages. Tasks of the interpreter and compiler, their role in programming.
Fundamentals of object and process-oriented programming languages, the OOP approach. Error handling, program testing.
Compulsory/Recommended Readings:
István Juhász: Programming Languages.,
http://www.tankonyvtar.hu/en/tartalom/tamop425/0046_programming_languages/index.html

Others

TTFBE2211
Classical mechanics

2+1 classes/week, 4+0 credit, E
Lecturer: Dr. Erdélyi Zoltán
Prerequisities: TTMBE0203

Motion of a point mass in one, two and three dimensions.
Compulsory/Recommended Readings:
David Halliday, Robert Resnick, Jearl Walker: Fundamentals of Physics, Wiley, 2013.

TTFBE2212
Theoretical mechanics
2+1 classes/week, 4+0 credit, E
Lecturer: Dr. Nagy Sándor
Prerequisities: TTFBE2201, TTMBE0206

Harmonic oscillation. Waves. Linear superposition and interference. General coordinates and constraints. Principle of least action. Euler—Lagrange equations. Symmetries, Galilei's principle of relativity, symmetry with respect to reflecion in space and time. Langrangians. Lagrange equations of the first kind. Symmetries and conservation laws, Noether's theorem. Newton's second law (force and force laws), action-reaction law, principle of independence of forces, momentum theorem, angular momentum theorem, mechanical equilibrium. Work—energy principle, potential energy, conservative force, conservation of energy, energy balance. Free motion, drag, static and kinetic friction. One-dimensional motion of a particle in a potential field. Simple harmonic oscillator, damped harmonic oscillator, driven harmonic oscillator, weak and strong damping, resonance. Hamilton equations, Legendre transformation. Model of an elastic medium, deformation and stress tensor, Hooke's law. Deformation of elastic media. Euler's description of fluid flow. Local law of conservation of matter. Hydrostatics. Euler's equation and Bernoulli's law.
Compulsory/Recommended Readings:
Herbert Goldstein: Classical Mechanics, Addison-Wesley, 1980.
L. D. Landau, E. M. Lifshitz: Course of Theoretical Physics, Volume 1 (Mechanics), Butterworth-Heinemann, 1976.
L. D. Landau, E. M. Lifshitz: Course of Theoretical Physics, Volume 6 (Fluid Mechanics), Butterworth-Heinemann, 1987.

TTTBE0030
European Union studies
1+0 classes/week, 1+0 credit, E
Lecturer: Dr. Teperics Károly
Prerequisities: none

Aim of the course: The objective of the course is to provide information about the theoretical background of integrations in general, the history of the European Union and its role in the world economy.
Topic: The process of reformation of the integration is going to be shown by the presentment of the institutions of the European Union. The process of enlargement, the characteristics of the fifth phase of the enlargement and the EU membership of Hungary is going to be emphasized especially.
Compulsory/Recommended Readings:
Farkas B., Várnay E. (2005): Bevezetés az Európai Unió tanulmányozásába, JATEPRESS Kiadó, Szeged
Palánkai T. (2004): Az európai integráció gazdaságtana, Aula Kiadó, Budapest
Horvath Z.: Kézikönyv az Európai Unióról – Akadémiai Kiadó, Budapest, 2005.

TTTBE0040
Basic environmental studies
1+0 classes/week, 1+0 credit, E
Lecturer: Dr. Nagy Sándor Alex
Prerequisities: none

Aims of the course: The student should acquire the more important natural science and social science connections of the based on ecology and focused on living organisms. The student have knowledge based on ecology and environmental elements of the environmental sciences. The student should be able to understand the necessity to recognise the sustainable development, knowing the history of environment protection and nature conservation.
The course involves: Environmental sciences and the ecological principles. Terminological

system of our environment. Environmental sciences and interdisciplinary. Challenge for science. The principle of precaution. Environmental problems. Natural environment. The surface of the Earth. Soil, the hydrosphere, the atmosphere.
The history of the natural conservation and the environmental protection; the sustainable development. Sustainable development. The economics of the human populations and the environmental sources. Limits of the growth. Human demography. The future of human populations. Resources and reserves. The soil as natural resource and the sustainable agriculture. The water supply and the water as power source. Biological resources. The effect of the human activity on the natural environment. The pollution of the atmosphere. Water pollution. The environmental pollution of industry. Technological forecast and the environment. Sustainable development: as a challenge

Compulsory/Recommended Readings:

Jackson, A,R.W., Jackson, J.M. 1996: Environmental Science. The natural environment and human impact. Longman, Singapore.
Brundtland, G.H. (Chair) 1987: Our common future. Oxford: Oxford University Press.
Cunningham, W.P. & Saigo, B.W. 1995: Environmental Science. A global concern. Dubuque: Wm.C. Brown Publishers.

Advanced prof module

TTMBE0109
Applied number theory
2+0 classes/week, 3+0 credit, E
Lecturer: Dr. Hajdu Lajos
Prerequisities: TTMBE0106

Basic notions of complexity theory. Some basic algorithms and their complexity. Approximation of real numbers by rationals, the theorem of Dirichlet. Liouville's theorem, a construction of transcendental numbers. Continued fractions and their properties. Finite and infinite continued fractions. Approximation with continued fractions. The LLL-algorihtm and some of its applications. Pseudoprimes and their properties. Carmichael-numbers and their role in probabilistic prime tests. Euler-pseudoprimes and their properties. The Soloway-Strassen probabilistic prime test. Strong pseudoprimes and their properties. The Miller-Rabin probabilistic prime test. Deterministic prime tests, Wilson's theorem, the description of the Agrawal-Kayal-Saxena test. The birthday paradox and Pollard's ρ–method. Fermat-factorization. Factorization with a factorbasis. Factorization with continued fractions.

Compulsory/Recommended Readings:
Neal Koblitz: A Course in Number Theory and Cryptography, Springer Verlag, 1994.
I. Niven, H. S. Zuckerman, H. L. Mongomery: An Introduction to the Theory of Numbers, Wiley, 1991.
Nigel Smart: The Algorithmic Resolution of Diophantine Equations, London Mathematical Society Student Text 41, Cambridge University Press, 1998.

TTMBG0110
Algorithms in algebra and number theory
0+3 classes/week, 0+3 credit, P
Lecturer: Dr. Tengely Szabolcs
Prerequisities: TTMBE0106

Linear algebra and applications using SageMath. Factoring polynomials over finite fields, the Berlekamp algorithm. Shamir's secret sharing algorithm.
Lattices, the LLL-algorithm and applications. Number theoretic functions in SageMath. Linear Diophantine equations, the Frobenious problem.
Conics and elliptic curves in SageMath.

Compulsory/Recommended Readings:
Victor Shoup: A Computational Introduction to Number Theory and Algebra, Cambridge University Press, 2005
William Stein: Elementary Number Theory: Primes, Congruences, and Secrets, Springer-Verlag, 2008

TTMBE0111, TTMBG0111
Introduction to cryptography
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Bérczes Attila
Prerequisities: TTMBE0109

Basic cryptographic concepts. Symmetric and asymmetric cryptosystems. The Cesar and the linear cryptosystems, DES, AES. The RSA cryptosystem and the analysis of its security. The discrete logarithm problem. Algorithms for solving the discrete logarithm problem. Cryptosystems based on the discrete logarithm problem. Elliptic curve cryptography. Basic cryptographic protocols. Digital signature. The basics of PGP.

Compulsory/Recommended Readings:
J. Buchmann: Einführung in die Kryptographie, Springer, 1999.
N. Koblitz: A Course in Number Theory and Cryptography, Springer, 1987.

TTMBE0209, TTMBG0209
Numerical analysis
3+2 classes/week, 4+2 credit, E+P
Lecturer: Dr. Fazekas Borbála
Prerequisities: TTMBE0102, TTMBE0203

The features of calculations with computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its versions: its algorithms, operational complexity, selection of pivot elements, non complete Gaussian elimination. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iteration methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximáció. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

Compulsory/Recommended Readings:

Atkinson, K.E.: Elementary Numerical Analysis. John Wiley, New York, 1993.

Lange, K.: Numerical analysis for statisticians. Springer, New York, 1999.

Press, W.H. – Flannery, B.P. – Tenkolsky, S.A. – Vetterling, W.T.: Numerical recipes in C. Cambridge University Press, Cambridge, 1988.

Engeln-Mullgens, G. – Uhling, F.: Numerical algorithms with C. Springer, Berlin, 1996.

TTMBG0210
Analysis with computer
0+3 classes/week, 0+3 credit, P
Lecturer: Dr. Fazekas Borbála
Prerequisities: TTMBE0203

The Maple; types of data, simple for-cycles, defining functions. Examination of functions; continuity, limit, zeros, extrema, constrained extrema. Differentiation, integration and numerical integration. Programming of simple quadrature rules. Solving differential equations with analytic methods and visualizing the solutions. Solving differential equations with numerical methods, programming Runge–Kutta fomulas. Ways of defining vectors and matrces. Vector and matrix operations, decompositions of matrices. Solving linear systems of equations with direct and iterative methods. Graphic tools in two dimension: making figures, rotation, reflection, parametrized curves. Graphic tools in three dimension: functions of two variables, space curves, surfaces, solid figures. Making animations, illustrating geometric and physical problems. Curve fitting; Lagrange and Hermite interpolation, Bezier curves and spline interpolation. For-cycle and while-cycle, conditional branches. Writing simple procedures: searching for primes, recursive functions, divisibility problems. Writing complex procedures: numerical differentiation and integration, approximation of functions, orthogonal polynomials and differential equations.

Compulsory/Recommended Readings:

W. Gander, J. Hrebicek: Solving Problems in Scientific Computing Using Maple and MATLAB. Springer-Verlag, Berlin, Heidelberg, New York, 1993, 1995.

TTMBE0211, TTMBG0211
Economic mathematics
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Mészáros Fruzsina
Prerequisities: TTMBE0204

Computation of future and present values, discounted present value and investment projects. Bounds for the budget, change of the budget line, consumer preferences, preference order. Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility. Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity. Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity. Production functions, marginal rate of substitution. CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function. Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly. Individual and social preferences, social welfare function. Arrow’s impossibility theorem. Consistent aggregation, bisymmetry equation. Influencing the distribution of incomes, the discounted present value of continuous income stream, Lorenz curve, Gini coefficient. Leontieff models.

Compulsory/Recommended Readings:

M. Carter: Foundations of Mathematical Economics, MIT Press, 2001.

K. Sydsaeter, P. Hammond, Mathematics for Economic Analysis, Pearson Publishing, 1995.

H. R. Varian: Intermediate Microeconomics: A Modern Approach, W.W. Norton, 1987.

TTMBG0308
Computer geometry
0+3 classes/week, 0+3 credit, P
Lecturer: Dr. Nagy Ábris
Prerequisities: TTMBE0302

Analytical tools of descriptive geometry: analytical geometry of projections, oblique and orthogonal axonometry, central projection, central axonometry. Curves and surfaces. Hermite, Bézier curves and surfaces, B-splines. Representation of polyhedra.

Compulsory/Recommended Readings:
A. Y. Brailov: Engineering Graphics, Springer International Publishing, 2016.
E. M. Mortensen: Geometric Modeling, Wiley Computer Publishing, 1997.

TTMBE0606, TTMBG0606
Algorithms
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Varga Nóra
Prerequisities: TTMBE0107

Classification of programming languages. Multi-character symbols. Data types. Instruction types. Cycles. Subprograms. The role of algorithms in computing. Functions, recursive functions. Probabilistic analysis. Randomized algorithms. Heap, heapsort. Quicksort. Sorting in linear time. Elementary data structures.
Compulsory/Recommended Readings:
T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction to Algorithms., MIT Press, Cambridge, 2009 (3rd ed.), http://www.realtechsupport.org/UB/SR/algorithms/Cormen_Algorithms_3rd.pdf
István Juhász: Programming Languages.,
http://www.tankonyvtar.hu/en/tartalom/tamop425/0046_programming_languages/index.html

TTMBE0607, TTMBG0607
Linear programming
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Mészáros Fruzsina
Prerequisities: TTMBE0102

Problems reducible to linear programming tasks. Extreme points of convex polyhedra, simplex algorithm and its geometry, sensitivity analysis. Duality. Transportation and assignment model, network models. Special linear programming models.
Compulsory/Recommended Readings:
Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.
Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

TTMBE0608, TTMBG0608
Nonlinear optimization
2+2 classes/week, 3+2 credit, E+P
Lecturer: Dr. Páles Zsolt
Prerequisities: TTMBE0204

Normed and Banach spaces. Spaces of linear and multilinear functionsi. Basic elements of differential calculus in normed spaces. Gateaux and Fréchet derivatives and their calculus. Strong and continuous Gateaux and Fréchet differentiability and relations between them. Inverse function theorem. The Fermat principle and Lagrange's multiplier theorem concerning extremum problems. Higher order Gateaux and Fréchet differentiability. Young's theorem and Taylor's theorem. Second order necessary and sufficient conditions for extremum problems. The first order basic problems of the calculus of variations with weak and strong extremum. Computing the derivative of functionals. Du Bois–Reymond lemma. Euler–Lagrange's first order necessary condition and second order necessary and sufficient conditions for weak extremum. The higher order basic problems of the calculus of variations and the Euler–Lagrange equation concerning them. Weierstrass's necessary and sufficient conditions for strong extremum.
Compulsory/Recommended Readings:
Dacorogna, B.: Introduction to the Calculus of Variations, Imperial College Press, London, 2014.
Durea, M.; Strugariu, R.: An Introduction to Nonlinear Optimization Theory, De Gruyter Open, Berlin, 2014.
Ioffe, A.D.; Tihomirov, V. M.: Theory of Extremal Problems, Studies in Mathematics and its Applications, 6.,

North-Holland Publishing Co., Amsterdam-New York, 1979.
Jahn, J.: Introduction to the Theory of Nonlinear Optimization, Springer Verlag, Berlin, 2007.

TTMBG0403
Computer statistcs
0+2 classes/week, 0+2 credit, P
Lecturer: Dr. Sikolya-Kertész Kinga
Prerequisities: TTMBE0401

Descriptive statistics, data visualisation, simulation techniques. Hypothesis testing, variance analysis, regression analysis.
<u>Compulsory/Recommended Readings:</u>
P. Dalgaard: Introductory Statistics with R. Springer, 2008.